STUDENT'S NAME: ______________________

TEACHER'S NAME: ______________________

PRO PATRIA

2018 | TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Assessment Task 4

Senior Examiner: Mr. S. Faulds

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- NESA-approved calculators may be used
- A reference sheet is provided for your use during the examination.
- In Questions 11–14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 2–4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 5–10)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section
- For each question, start a new answer booklet.

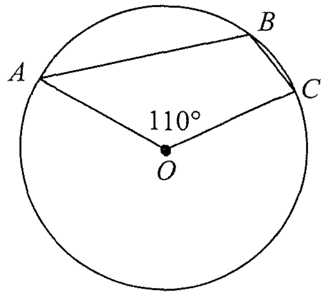
Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2018 HSC Mathematics Extension 1 Examination.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10. This answer sheet is attached to the back of your examination paper. It may be removed and handed in with your answer booklets for Section 2.

1.



In the diagram above, points A, B and C lie on the circumference of the circle, centre O. What is the size of $\angle ABC$?

A: 70° **B:** 110° **C:** 125° **D:** 250°

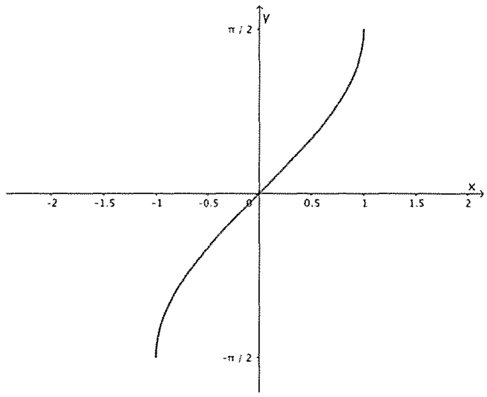
2. What is the value of $\lim\limits_{x\to 0}\dfrac{2\sin 5x}{3x}$?

A: $\dfrac{10}{3}$ **B:** $\dfrac{6}{5}$ **C:** $\dfrac{2}{3}$ **D:** 0

3. Which expression below is equal to $\displaystyle\int \frac{x}{x^2+1}\,dx$?

A: $\dfrac{1}{2}\ln(x^2+1)+c$ **B:** $\ln(x^2+1)+c$ **C:** $2\ln(x^2+1)+c$ **D:** $\ln x+\dfrac{x^2}{2}+c$

4. Consider the function $f(x) = \sin^{-1}x$, the graph of which is shown below.



What is the domain of the function?

A: $-\pi \le y \le \pi$ **B:** $-1 \le x \le 1$ **C:** $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ **D:** $-\frac{1}{2} \le x \le \frac{1}{2}$

5. What is the remainder when $P(x) = 2x^3 - 6x^2 + 4x + 3$ is divided by $2x - 1$?

A: 3 **B:** -9 **C:** $\frac{15}{4}$ **D:** $-\frac{3}{4}$

6. A curve is defined by the parametric equations $x = \sin 2t$ and $y = \cos 2t$. Which of the following, in terms of t, equates to $\frac{dy}{dx}$?

A: $-\tan 2t$ **B:** $2\cos 2t$ **C:** $2\sin 4t$ **D:** $-\cot 2t$

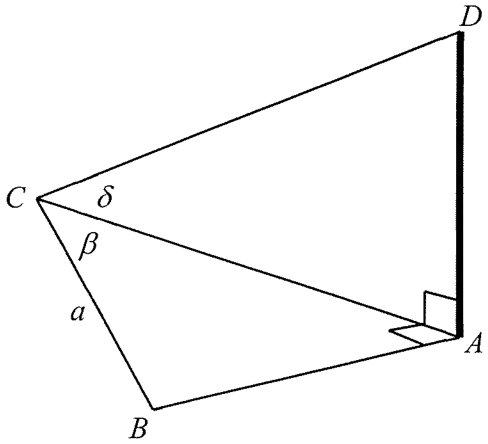
7. A mathematics department consists of 5 female and 5 male teachers. How many committees of three teachers can be chosen which contain at least one female and at least one male?

A: 100 **B:** 120 **C:** 200 **D:** 250

8. Research into Alzheimer's disease suggests that the rate of loss of percentage brain function is proportional to the percentage brain function already lost. A particular Alzheimer's disease patient was initially diagnosed with a 20% loss of brain function.
If L is the percentage brain function lost and k is a constant, which of the following equations represents the loss of percentage brain function for this particular patient?

A: $L = ke^{0.2t}$ **B:** $L = ke^{20t}$ **C:** $L = 20e^{kt}$ **D:** $L = 80e^{kt}$

9. In the figure, ABC is a triangle on a horizontal plane and AD is a vertical flag pole. If $BC = a$, which of the following expression is equal to AD?



A: $a\sin(\beta + \delta)$ **B:** $a\cos\beta\,\sin\delta$ **C:** $a\cos\beta\,\tan\delta$ **D:** $a\sin\beta\,\tan\delta$

10. The letters of the word TWITTER are arranged randomly. How many of these arrangements would have all three T's seperated?

A: 240 **B:** 480 **C:** 720 **D:** 1440

Section II starts on the next page.

Section II

90 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer each question in a new answer booklet.

All necessary working should be shown in every question.

Question 11 (15 marks) **Start a new answer booklet.** **Marks**

(a) Show that the acute angle between the lines $x-2y=0$ and $3x-y-15=0$ is $\frac{\pi}{4}$ radians. **2**

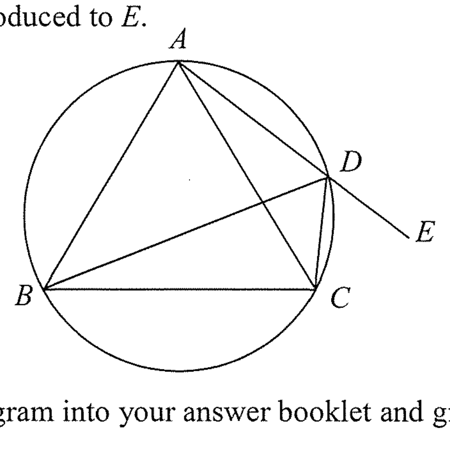
(b) Solve the inequality $\frac{x+1}{x-5} \le 0$. **2**

(c) If $t=\tan\frac{\theta}{2}$, show that $\frac{\sin 2\theta(1-\cos\theta)}{\cos\theta(1-\cos 2\theta)}=t$. **3**

(d) **(i)** Explain why the expression $\frac{k+1}{2}$ is an integer if k is an odd integer. **1**

(ii) Prove by mathematical induction that the expression n^2-1 is divisible by 8 for all **odd** integral values of n. **3**

(e) In the diagram below, D is a point on the minor arc AC of the circle passing through A, B and C. AD is produced to E.



(i) Copy the diagram into your answer booklet and give a reason why $\angle CDE = \angle ABC$. **1**

(ii) Hence, show that if $BC = AC$, then DC bisects $\angle BDE$. **3**

Question 12 (15 marks) **Start a new answer booklet.** **Marks**

(a) Let the cubic polynomial, $P(x) = x^3 - 3x^2 - 4x + 12$ have the roots α, β and γ .

(i) Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. **2**

(ii) Given that two of its roots have a sum of zero, find the values of α, β and γ. **2**

(b) The function $f(x) = e^{x^2} - x - 3$ has a zero near $x = 1.1$. **3**
Using one application of Newton's Method, find a better approximation for the zero correct to 2 decimal places.

(c) Consider the expansion $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + ... + \binom{n}{n}x^n$.

(i) By first integrating both sides of the above expansion show that: **3**

$$\binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + ... + \frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1} - 1}{n+1}$$

(ii) Hence show that: **2**

$$\frac{1}{2}\binom{n}{1} + \frac{2}{3}\binom{n}{2} + \frac{3}{4}\binom{n}{3} + ... + \frac{n}{n+1}\binom{n}{n} = \frac{2^n(n-1)+1}{n+1}$$

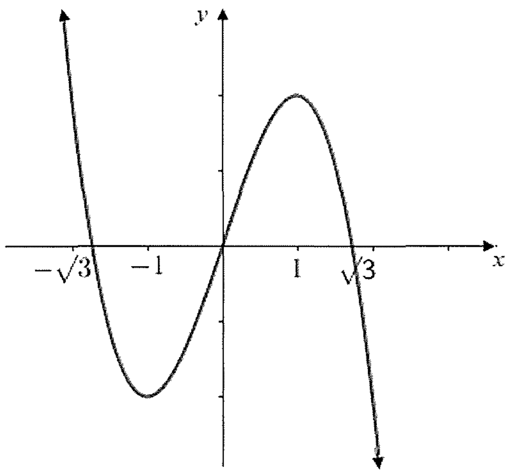
(d) The point $P(2t, t^2)$ lies on the parabola with equation $x^2 = 4y$ and focus at F. **3**
The point M divides the interval FP externally in the ratio 3:1.
Show that the co-ordinates of M are $x = 3t$ and $y = \frac{1}{2}(3t^2 - 1)$.

Question 13 (15 marks) **Start a new answer booklet.** **Marks**

(a) Use the substitution $u = x + 2$ to find $\int x(x+2)^{99}\,dx$. **2**

(b) The gradient at any point (x, y) of a curve is given by $\dfrac{dy}{dx} = \cos^2 x$. If the curve passes through the point $\left(\dfrac{\pi}{2}, \pi\right)$, find its equation. **3**

(c) The diagram shows the curve of $y = 3x - x^3$.



(i) Find the largest domain containing the origin for which $f(x)$ has an inverse function, $f^{-1}(x)$. **1**

(ii) State the domain of $f^{-1}(x)$. **1**

(iii) Find $\dfrac{dx}{dy}$ of the function, $f^{-1}(x)$. **2**

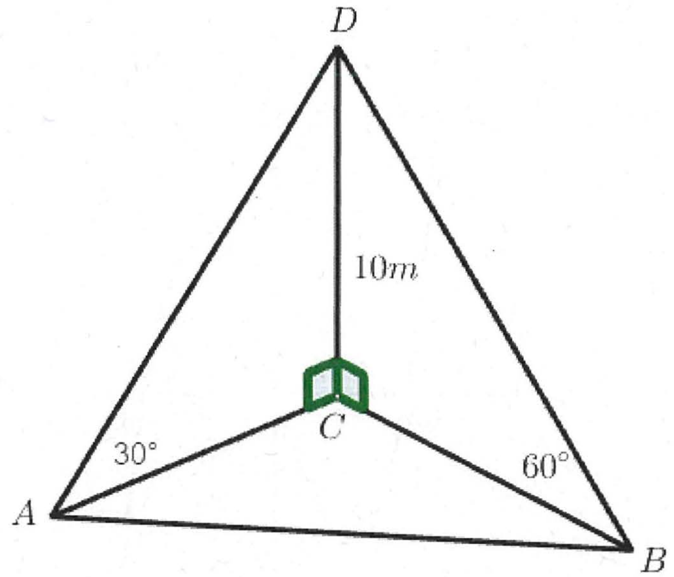
(d) **(i)** Differentiate $x\sin^{-1}x + \sqrt{1-x^2}$. **2**

(ii) Hence, evaluate $\int_0^1 \sin^{-1} x\,dx$. **1**

Question 13 continues on the next page...

Question 13 (continued) **Marks**

(e) In the diagram, CD is a vertical pole 10 m high. CB is the shadow of the pole when the elevation of the sun is 60° and CA is the shadow of the pole when the elevation of the sun is 30°. A, B and C are all on level ground.



(i) Show that $BC = 10 \cot 60°$. **1**

(ii) Hence, find the distance of AB if $\angle ACB = 60°$. **2**
(Give your answer to 3 significant figures.)

Question 14 (15 marks) **Start a new answer booklet.** **Marks**

(a) A bowl of soup is cooling in a room that has a constant temperature of $20°$. At time t, measured in minutes, the temperature, T, of the soup is decreasing according to the differential equation:

$$\frac{dT}{dt} = -k(T-20)$$

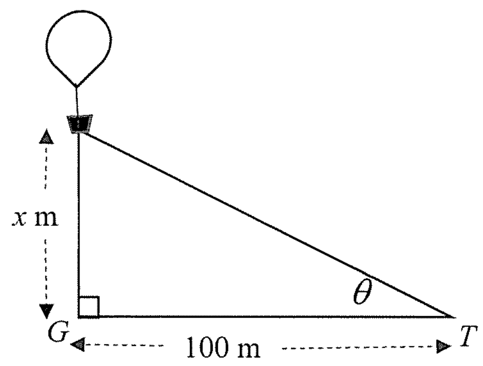
where k is a positive constant. The initial temperature of the soup is $100°$ and it cools to $70°$C after 5 minutes.

(i) Verify that $T = 20 + Ae^{-kt}$ is a solution to the differential equation, where A is a constant. **1**

(ii) Find the values of A and k. **2**

(iii) Find the temperature of the soup after 15 minutes, Give your answer to the nearest degree. **1**

(b)



Themba is watching a weather balloon being released from a point G which is 100 meters away on horizontal ground. The weather balloon is rising vertically at a constant rate of 5 m/s. This information is illustrated on the diagram above. **3**

Let θ radians be the angle of elevation of the weather balloon at time t seconds and let x metres be the distance the weather balloon has travelled in that time.

Find the rate of change of the angle of elevation of the weather balloon, when $\theta = \frac{\pi}{4}$.

Question 14 continues on the next page...

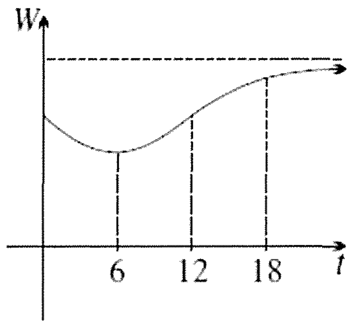
(c) The rate of descent of a submarine, from the surface, into the ocean is given as: **2**

$$\frac{dh}{dt}=1-\left(1+t\right)^{-2}$$

where h is the depth of the submarine in metres and t the time in seconds.

Find the depth of the submarine after 1 minute, correct to one decimal place.

(d)



The graph above shows the average weight W of a herd of beef cattle over a period of time t, where t is in months. After a period of drought, the average weight of the herd stabilised. **2**

Sketch the graph of the rate, $\dfrac{dW}{dt}$ at which W was changing over this period.

(e) A group of friends, made up of four females and three males are to be seated in consecutive seats at a concert. If seat numbers are allocated randomly, what is the probability that exactly three of the girls will be sitting next to one another? **2**

(f) The probability of a certain brand of light globe being faulty is 0.9%. In a batch of 8 such light globes, find the probability, as a percentage to one decimal place, that at least 7 of the light globes will work. **2**

END OF EXAMINATION

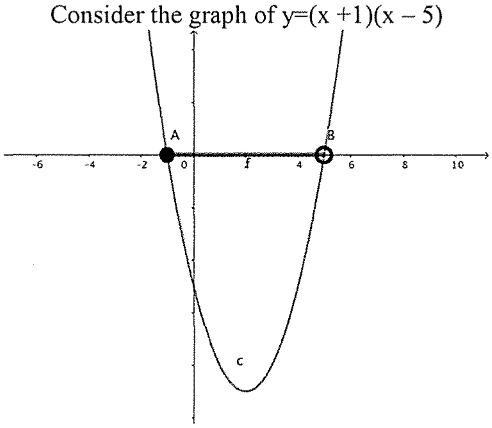
Year 12 Mathematics Trial HSC Examination 2018		
Multiple Choice Questions		
	Solutions	
	(see answer table below)	**1 mark for each correct solution**

1	**C**
2	**A**
3	**A**
4	**B**
5	**C**
6	**A**
7	**A**
8	**C**
9	**C**
10	**A**

Question No. 11 Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE2 uses multi-step deductive reasoning in a variety of contexts
HE2 uses inductive reasoning in the construction of proofs

Outcome	Solutions	Marking Guidelines
PE2	**(a)** $x-2y=0 \quad \Rightarrow m_1=\frac{1}{2}$ $3x-y-15=0 \Rightarrow m_2=3$ $\tan\alpha=\left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$ $=\left\lvert\frac{\frac{1}{2}-3}{1+\frac{3}{2}}\right\rvert$ $=\left\lvert\frac{-\frac{5}{2}}{\frac{5}{2}}\right\rvert$ $=\lvert-1\rvert$ $=1$ $\therefore \alpha=\frac{\pi}{4}$ radians (as required)	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution including finding the gradient of the two lines.
PE2	**(b)** $\frac{x+1}{x-5}\le 0, \qquad x\ne 5$ Multiplying both sides by $(x-5)^2$ $(x+1)(x-5)\le 0, \qquad x\ne 5$ Consider the graph of y=(x +1)(x – 5)  Function values will be less than zero when: $-1\le x\le 5$, but $x\ne 5$ $\therefore$ Solution: $-1\le x<5$	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.

HE2	**(c)** $$\text{L.H.S.} = \frac{\sin 2\theta(1-\cos\theta)}{\cos\theta(1-\cos 2\theta)}$$ $$= \frac{2\sin\theta\cos\theta(1-\cos\theta)}{\cos\theta(1-(\cos^2\theta-\sin^2\theta))}$$ $$= \frac{2\sin\theta(1-\cos\theta)}{(\sin^2\theta+\sin^2\theta)}$$ $$= \frac{2\sin\theta(1-\cos\theta)}{2\sin^2\theta}$$ $$= \frac{(1-\cos\theta)}{\sin\theta}$$ $$= \frac{1-\frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}}$$ $$= \frac{1+t^2-1+t^2}{2t}$$ $$= \frac{2t^2}{2t}$$ $$= t$$ $$= \text{R.H.S}$$	**3 marks** Correct solution showing all required steps. **2 marks** Substantial progress towards correct solution, showing most of the required steps. **1 mark** Some progress towards a correct solution.
HE2	**(d) (i)** If k is odd then $k + 1$ must be even. If an even number is divided by 2 ie. $\frac{k+1}{2}$, there is no remainder and hence, the result is an integer.	**1 mark** Correct explanation.
HE2	**(ii)** Prove true for $n = 1$ $n^2 - 1 = 1 - 1$ $= 0$ which is divisible by 8. Hence, true for $n = 1$. Assume true for $n = k$ where k is odd ie. assume: $k^2 - 1 = 8M$, where M is an integer Prove true for $n = k + 2$, where $k + 2$ is the next odd integer. ie. Prove $[(k+2)^2 - 1]$ is divisible by 8 $[(k+2)^2 - 1] = k^2 + 4k + 4 - 1$ $= k^2 - 1 + 4k + 4$ $= 8M + 4k + 4$ $= 8\left(M + \frac{k+1}{2}\right)$ which is divisible by 8 since $\left(M + \frac{k+1}{2}\right)$ is an integer. If the result is true for $n = k$ it is also true for $n = k + 2$. Since the result is true for n = 1 it is also true for n = 1 + 2 = 3, n = 3 + 2 = 5, etc. Hence, by the process of mathematical induction, the result is true for all positive odd integers.	**3 marks** Correct solution showing all required steps. **2 marks** Substantial progress towards correct solution. **1 mark** Some progress towards a correct solution. Including showing the result is true for $n = 1$.

HE2	**(e) (i)** $\angle CDE = \angle ABC$ since the exterior angle of a cyclic quadrilateral ($ABCD$) is equal to the interior opposite angle.	**1 mark** Correct reason.
HE2	**(ii)** Let $\angle CDE = \theta$ $\therefore \quad \angle ABC = \theta \qquad$ (shown above) Now, since $BC = AC$ $\angle BAC = \angle ABC \qquad$ (angles standing on equal chords) $= \theta$ Also, $\angle BDC = \angle BAC$ (angles standing on same chord) $= \theta$ Hence, $\angle BDC = \angle CDE = \theta$ $\therefore DC$ bisects $\angle BDE$	**3 marks** Correct solution showing all reasoning. **2 marks** Substantial progress towards correct solution showing relevant reasoning. **1 mark** Some progress towards correct solution.

Year 12 Mathematics Extension 1 Half Yearly Examination 2005		
Question No. 12 Solutions and Marking Guidelines		
Outcomes Addressed in this Question		
HE2 uses inductive reasoning in the construction of proofs		
Outcome	**Solutions**	**Marking Guidelines**
HE2	**(a)(i)** $\alpha\beta\gamma=-12$ $\alpha\beta+\alpha\gamma+\beta\gamma=-4$ $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma}$ $=\frac{-4}{-12}$ $=\frac{1}{3}$	2 marks : correct solution 1 marks : substantial progress towards correct solution
HE2	**(ii)** Let $\alpha+\beta=0$ $\alpha=-\beta$ $\alpha+\beta+\gamma=3$ $\therefore\ \gamma=3$ $\alpha\beta\gamma=-12$ $-3\beta^2=-12$ $\beta^2=4$ $\beta=\pm 2$ $\therefore$ Roots are 2, -2 and 3	2 marks : correct solution 1 marks : substantial progress towards correct solution
HE2	**(b)** $f(x)=e^{x^2}-x-3$ $f'(x)=2xe^{x^2}-1$ $\therefore x_1=1.1-\frac{e^{1.21}-1.1-3}{2.4e^{1.21}-1}$ $=1.21705.....$ $\therefore\ x_1=1.22$	3 marks : correct solution 2 marks: substantial progress towards correct solution 1 mark : significant progress towards correct solution

HE2	**(c)(i)** $(1+x)^n=\binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+..........+\binom{n}{n}x^n$ Integrating both sides: $\frac{(1+x)^{n+1}}{n+1}=\binom{n}{0}x+\frac{1}{2}\binom{n}{1}x^2+\frac{1}{3}\binom{n}{2}x^3+..........+\frac{1}{n+1}\binom{n}{n}x^{n+1}+c$ Let $x=0$, $\therefore c=\frac{1}{n+1}$. $\therefore \frac{(1+x)^{n+1}}{n+1}-\frac{1}{n+1}=\binom{n}{0}x+\frac{1}{2}\binom{n}{1}x^2+\frac{1}{3}\binom{n}{2}x^3+..........+\frac{1}{n+1}\binom{n}{n}x^{n+1}$ Let $x=1$, $\frac{2^{n+1}}{n+1}-\frac{1}{n+1}=\binom{n}{0}+\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}+..........+\frac{1}{n+1}\binom{n}{n}$ $\therefore \frac{2^{n+1}-1}{n+1}=\binom{n}{0}+\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}+..........+\frac{1}{n+1}\binom{n}{n}$	3 marks : correct solution 2 marks: substantial progress towards correct solution 1 mark : significant progress towards correct solution
HE2	**(ii)** $\binom{n}{0}+\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}+..........+\frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1}-1}{n+1} \quad .... (1)$ Let $x=1$ in the original expansion $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+..........\binom{n}{n} = 2^n \quad .... (2)$ $(2)-(1)$: $\frac{1}{2}\binom{n}{1}+\frac{2}{3}\binom{n}{2}+..........+\left(1-\frac{1}{n+1}\right)\binom{n}{n} = 2^n-\frac{2^{n+1}-1}{n+1}$ $\frac{1}{2}\binom{n}{1}+\frac{2}{3}\binom{n}{2}+..........+\left(\frac{n}{n+1}\right)\binom{n}{n} = \frac{2^n\times n+2^n-2^{n+1}+1}{n+1}$ $\frac{1}{2}\binom{n}{1}+\frac{2}{3}\binom{n}{2}+..........+\left(\frac{n}{n+1}\right)\binom{n}{n} = \frac{n2^n+2^n-2\times 2^n+1}{n+1}$ $\frac{1}{2}\binom{n}{1}+\frac{2}{3}\binom{n}{2}+..........+\left(\frac{n}{n+1}\right)\binom{n}{n} = \frac{n2^n-2^n+1}{n+1}$ $\therefore \frac{1}{2}\binom{n}{1}+\frac{2}{3}\binom{n}{2}+..........+\left(\frac{n}{n+1}\right)\binom{n}{n} = \frac{2^n(n-1)+1}{n+1}$	2 marks : correct solution 1 marks : substantial progress towards correct solution

HE2	**(d)** $x^2 = 4ay \quad \rightarrow \quad x^2 = 4ay$ $\therefore\ a = 1$ and focus is $(0,\ 1)$ $P(2t,\ t^2)$ Divide externally 3:1 $\therefore\ m:n = 3:-1$ $\therefore\ x = \dfrac{mx_2 + nx_1}{m+n}$ and $y = \dfrac{my_2 + ny_1}{m+n}$ $x = \dfrac{3\times 2t - 1\times 0}{3-1}$ $\quad y = \dfrac{3\times t^2 - 1\times 1}{3-1}$ $\therefore\ x = 3t$ $\quad \therefore\ y = \dfrac{3t^2 - 1}{2}$	3 marks : correct solution 2 marks: substantial progress towards correct solution 1 mark : significant progress towards correct solution

Outcomes Addressed in this Question

PE1 Appreciates the role of mathematics in the solution of practical problems
HE4 use the relationship between functions, inverse functions and their derivatives
HE6 determines integrals by reduction to a standard form through a given substitution

Outcome	Solutions	Marking Guidelines
HE6	**(a)** Let $u = x + 2 \qquad x = u - 2$ $du = dx$ $\int x(x+2)^{99}\,dx$ $= \int (u-2)u^{99}du$ $= \int \left(u^{100} - 2u^{99}\right)du$ $= \frac{u^{101}}{101} - \frac{2u^{100}}{100} + C$ $= \frac{(x+2)^{101}}{101} - \frac{(x+2)^{100}}{50} + C$	**2 marks** for correct solution **1mark** for substantial progress towards solution
HE4	**(b)** $y = \int \cos^2 x dx$ $= \int \frac{1}{2}(1+\cos 2x)xdx$ $= \frac{1}{2}x + \frac{1}{2}\frac{\sin 2x}{2} + C$ The curve passes through the point $\left(\frac{\pi}{2}, \pi\right)$ $\pi = \frac{1}{2}\left(\frac{\pi}{2}\right) + \frac{\sin 2}{4}\left(\frac{\pi}{2}\right) + C$ $\pi = \left(\frac{\pi}{4}\right) + 0 + C$ $C = \pi - \left(\frac{\pi}{4}\right) = \frac{3\pi}{4}$ Hence, the equation is: $y = \frac{1}{2}x + \frac{\sin 2x}{4} + \frac{3\pi}{4}$	**3 marks** for correct solution **2 marks** for substantial progress towards solution **1 mark** for limited progress towards solution
	(c)(i) $-1 \le x \le 1$	**1 mark** for correct solution
	(ii) When $x = 1,\ y = 3(1) - 13 = 2$ $x = -1,\ y = 3(-1) - 13 = -2$ domain of $f^{-1}(x)$ is $-2 \le x \le 2$	**1 mark** for correct solution

PE1	**(iii)** $y = 3x - x^3$ Inverse $x = 3y - y^3$ $\frac{dx}{dy} = 3 - 3y^2$	**2 marks** for complete correct solution **1 mark** for substantial progress that could lead to a correct explanation
	(d)(i) $y = x\sin^{-1}x + \sqrt{1-x^2}$ $\frac{dy}{dx} = x \times \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \times 1 + \frac{1}{2}\left(1-x^2\right)^{\frac{-1}{2}}(-2x)$ $= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \left(1-x^2\right)^{\frac{-1}{2}}(x)$ $= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \frac{x}{\sqrt{1-x^2}}$ $= \sin^{-1}x$	**2 marks** for correct solution **1 mark** for substantial progress that could lead to a correct explanation
	(ii) $\int_0^1 \sin^{-1}x\,dx = \left[x\sin^{-1}x + \sqrt{1-x^2}\right]_0^1$ $= \sin^{-1}(1) - 1$ $= \frac{\pi}{2} - 1$	**1 mark** for correct solution
	(e) (i) In ΔDCB $\tan 60^\circ = \frac{10}{BC}$ $\therefore BC = \frac{10}{\tan 60^\circ} = 10\cot 60^\circ$ Similarly for ΔDAC, $AC = 10\cot 30^\circ$	**1 mark** for correct solution
	(ii) In ΔDAC, $AB^2 = AC^2 + BC^2 - 2AC.BC\cos 60^\circ$ $= (10\cot 30^\circ)^2 + (10\cot 60^\circ)^2 - 2(10\cot 30^\circ)(10\cot 60^\circ)\cos 60^\circ$ $= \left(10\sqrt{3}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 - 2\times 10\sqrt{3} \times \frac{10}{\sqrt{3}} \times \cos 60^\circ$ $= 300 + \frac{100}{3} - 200 \times \frac{1}{2}$ $= \frac{700}{3}$ $\therefore AB = 15\cdot 275$ $= 15.3\,\text{m}$	**2 marks** for correct solution **1mark** for substantial progress towards solution

Outcomes Addressed in this Question

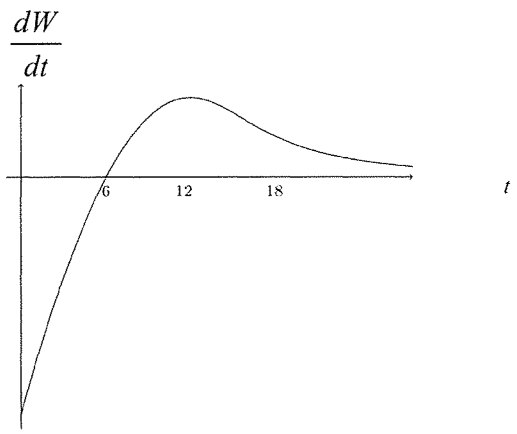
PE3 solves problems involving **permutations and combinations**, inequalities, polynomials, circle geometry and parametric representations

HE3 uses a variety of strategies to investigate mathematical models of situations involving **binomial probability**, projectiles, simple harmonic motion, or **exponential growth and decay**

HE5 applies the chain rule to problems including those involving velocity and acceleration as functions of displacement

HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form

Outcome	Solutions	Marking Guidelines
HE3	(a)(i) $T = 20 + Ae^{-kt}$ $\frac{dT}{dt} = A.-k\,e^{-kt}$ $\therefore \frac{dT}{dt} = -k.Ae^{-kt}$ $\therefore \frac{dT}{dt} = -k(T-20)$	**1 mark**: correct solution
HE3	(ii) When $t = 0$, $T = 100$ using $T = 20 + Ae^{-kt}$, $100 = 20 + Ae^{0}$ $\therefore A = 80$ $\therefore T = 20 + 80e^{-kt}$ When $t = 5$, $T = 70$, $70 = 20 + 80e^{-5k}$ $\frac{5}{8} = e^{-5k}$ $1.6 = e^{5k}$ $5k = \ln 1.6$ $\therefore k = \frac{1}{5}\ln 1.6$	**2 marks**: A & k correct **1 mark** : one of above correct
HE3	(iii) $T = 20 + 80e^{-\frac{1}{5}(\ln 1.6)t}$ When $t = 15$, $T = 20 + 80e^{-3\ln 1.6}$ $= 40°$ (to nearest degree)	**1 mark** : correct answer
HE5 **HE7**	(b) $\tan\theta = \frac{x}{100}$, $\therefore x = 100\tan\theta$ $\frac{dx}{d\theta} = 100\sec^2\theta = \frac{100}{\cos^2\theta}$ $\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt}$ $\therefore \frac{d\theta}{dt} = \frac{\cos^2\theta}{100} \times 5 = \frac{\cos^2\theta}{20}$ When $\theta = \frac{\pi}{4}$, $\frac{d\theta}{dt} = \frac{\cos^2\left(\frac{\pi}{4}\right)}{20} = \frac{1}{40}$ $\therefore$ at this instant, angle changing at $\frac{1}{40}$ radians/sec	**3 marks** : correct solution **2 marks**: substantial progress towards correct solution **1 mark** : significant progress towards correct solution

HE7	(c) $\frac{dh}{dt}=1-(1+t)^{-2}$ $h=\int 1-(1+t)^{-2}dt$ $\therefore h=\left[t-\frac{(1+t)^{-1}}{-1}\right]_0^{60}$ $\therefore h=\left[t+\frac{1}{1+t}\right]_0^{60}$ $\therefore h=\left[t+\frac{1}{1+t}\right]_0^{60}$ = 59.0 m	**2 marks** : correct solution **1 marks** : substantial progress towards correct solution
HE7	(d)  	**2 marks** : correct solution **1 marks** : substantial progress towards correct solution
PE3	(e) Total number of ordered arrangements: 7! Number of groups of different groups of 3 girls: $^4C_3$ Tie the 3 girls together as *X*: 3! Ordered ways As this group of 3 girls can not be seated next to the other Girl, seat the 3 boys first, creating gaps between them: 3! ways of seating the boys _ B _ B _ B _ Group X have 4 spots they could go in, then the other girl has 3 spots to go in. $\therefore$ number of ordered arrangements with exactly three of the girls together is $\therefore\ ^4C_3\times 3!\times 3!\times 4\times 3=1728$ $\therefore$ required probability is $\frac{1728}{7!}=\frac{12}{35}$	**2 marks** : correct solution **1 mark** : substantial progress towards correct solution
HE3	(f) At least 7 faulty = 7 work or 8 work P(work) = 99.1% P(not work) = 0.9% P(at least 7) = P(exactly 7) + P(exactly 8) $= {}^8C_7(99.1\%)^7(0.9\%)^1+{}^8C_8(99.1\%)^8(0.9\%)^0$ = 0.9978 (to 4 dp)	**2 marks** : correct solution **1 mark** : substantial progress towards correct solution